

Enabling mixed-precision with VerifiCarlo: Sharing CEEC experience

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Outline

Energy efficiency in computing

Methodology for energy-efficient computing

Mixed-precision Nekbone and Neko

Energy efficiency: from hardware to NLA

- ▶ Supercomputing is constrained by **power consumption**
- Power-efficient hardware
 - ▶ RIKEN's Fugaku w A64FX (FP64:FP32:FP16 = 1:2:4)
 - ▶ EPI (ARM, FPGA, RISC-V)
 - ▶ **Jülich to host the first EPI-based supercomputer**
- ▶ Numerical linear algebra is known to be dominant by **double precision**
- **Energy-efficient algorithms**
 - math** Mixed and adaptive precision computing
 - code** Communication hiding or avoiding
 - tools** Numerical abnormalities and precision cropping

Measuring energy consumption



Centre of Excellence in Exascale CFD

Best Practice Guide

Harvesting energy consumption on
European HPC systems: Sharing
Experience from the CEEC project

lakymchuk et al. Zenodo, 2024
doi:10.5281/zenodo.13306639

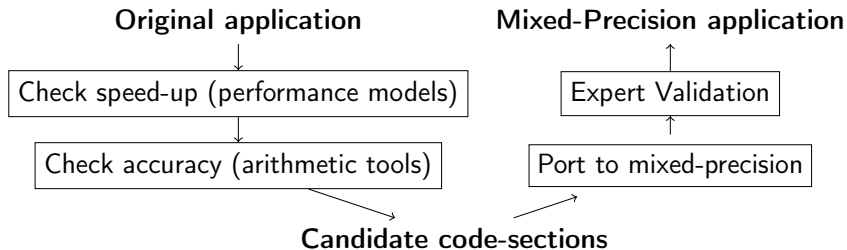
- ▶ **More complex than measuring time-to-solution**
- ▶ Measurements require elevated privileges

Objectives

- ▶ Facilitate energy measurements on the European HPC systems
- ▶ Teach the community how to conduct such measurements
- ▶ Provide examples with easy-to-use guide

Methodology

Methodology to enable mixed-precision algorithmic solutions in applications with accuracy guarantees.



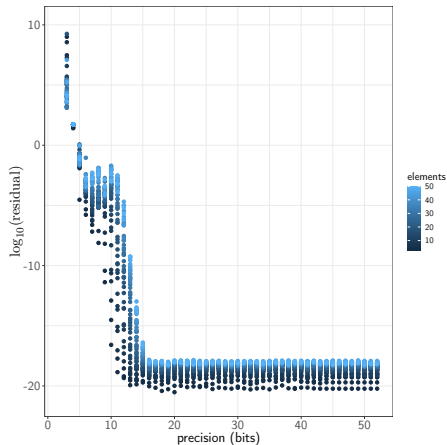
Nekbone w Vprec

$$Ax = b$$

while ($\tau > \tau_{\max}$)

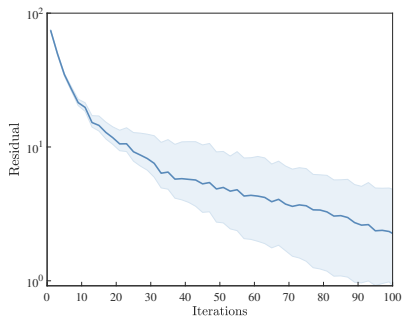
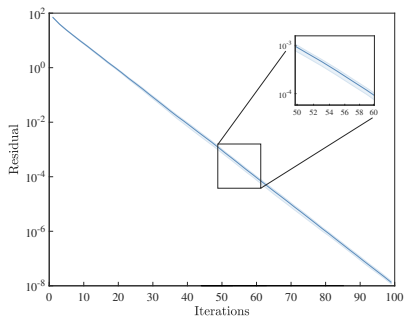
Step	Operation
$S1 :$	$w := Ad$
$S2 :$	$\rho := \beta / \langle d, w \rangle$
$S3 :$	$x := x + \rho d$
$S4 :$	$r := r - \rho w$
$S5 :$	$z := M^{-1}r$
$S6 :$	$\beta := \langle z, r \rangle$
$S7 :$	$d := (\beta / \beta_{old})d + z$
$S8 :$	$\tau := \langle r, r \rangle$

end while



Nekbone w MCA for FP32

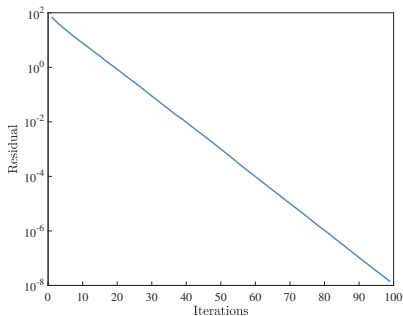
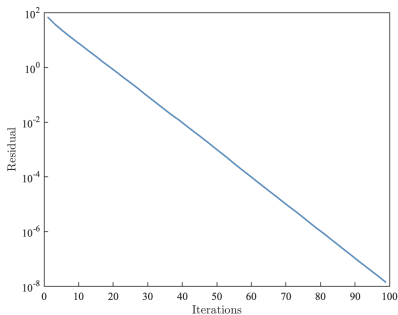
Entire program, no preconditioner



- ▶ Random Rounding (*rr*) mode (left)
- ▶ MCA (*mca*) mode (right)
- ▶ Issue in initialization $10^9 \cos(x) \rightarrow$ focus on the solver only

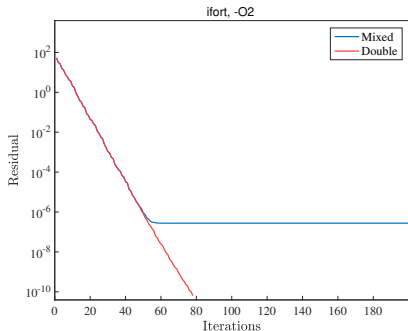
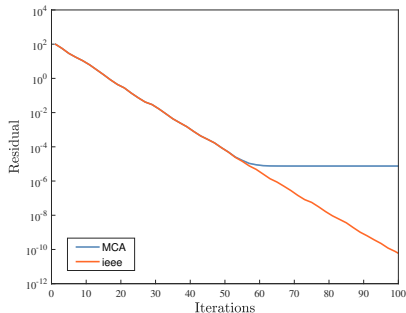
Nekbone w MCA for FP32

Only the CG loop, no preconditioner



- ▶ Random Rounding (*rr*) mode (left)
- ▶ MCA (*mca*) mode (right)

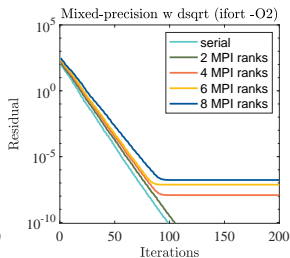
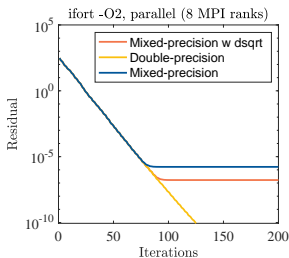
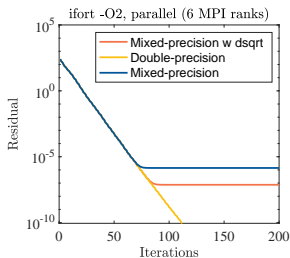
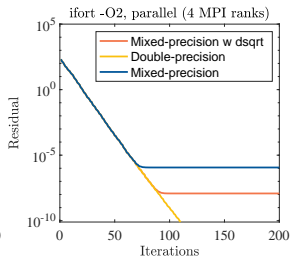
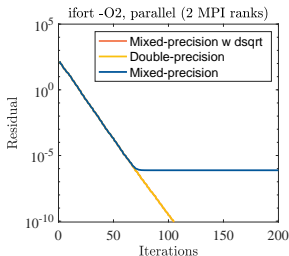
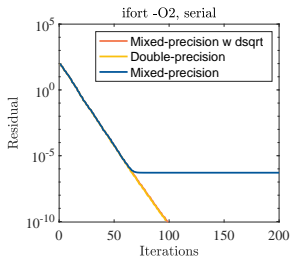
Nekbone: CG with preconditioner



- ▶ Verifcarlo predicts stagnation (left plot)
- ▶ Stagnation in the mixed-precision Nekbone (right plot)
- ▶ Stagnation occurs after 61st iteration w residual $r = 7.94 \times 10^{-6}$

Nekbone: CG with preconditioner

Square root



Nekbone: CG with preconditioner

Mixed-precision strategies

Mixed-precision strategy evaluation							
GSO mode	MPI	Precond.		PCG		Conv.	
		Ops	GSO	Ops	GSO		
pairwise	< 4	fp32	fp32			stagnates	
		fp64	fp64		fp32	converges	
		fp32+fp64sqrt	fp32			converges	
		fp32	fp64			converges	
	≥ 4	fp64	fp64	fp32			stagnates
		fp32+fp64sqrt	fp32		fp32		stagnates
		fp32	fp64				stagnates
		fp32+fp64sqrt	fp64		fp64		converges
		fp64	fp64		fp64		converges

- ▶ Gather-Scatter Operations have strong impact on convergence
- ▶ dot product impacts convergence too

Nekbone: CG with preconditioner

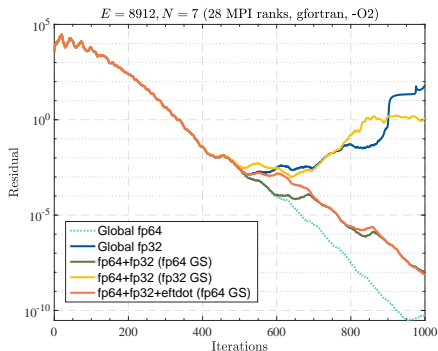
128 elements per MPI rank on MareNostrum 5: Intel Sapphire Rapids 8460Y+ w 40 cores

MPI ranks/ secs	8	20	40	80
Double	5.79	8.98	13.33	24.02
Mixed-1 ^a	4.79	5.99	9.42	14.85
Gain-1	17.27%	33.30%	29.33%	38.18%
Mixed-2 ^b	4.88	6.02	8.37	10.11
Gain-2	15.72%	32.96%	37.21%	2.38x

MPI ranks/ joules	8	20	40	80
Double	1875	3035	7191	16261
Mixed-1 ^a	939	2088	3133	7428
Gain-1	1.97x	31.20%	2.30x	2.19x
Mixed-2 ^b	934	1566	2570	3482
Gain-2	2.01x	1.94x	2.80x	4.67x

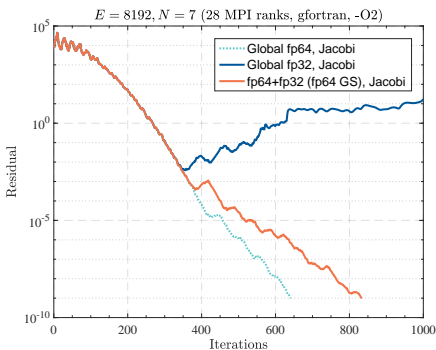
^a fp64+fp32+fp64sqrt (fp32 GS, allreduce)^b fp64+fp32+fp64sqrt (fp64 GS, pairwise)

Neko: convergence



CG with the identity preconditioner

- ▶ Solving the Poisson's equation with Neko
- ▶ The winning strategy is fp64+fp32 (fp64 GS)



CG with the Jacobi preconditioner

Neko: CG with the identity preconditioner

16,384 elements with pol. degree 7 on MareNostrum 5

MPI ranks/ secs	8	20	40	80
Double	14.10	6.61	4.80	1.87
Mixed	12.50	5.77	3.41	1.55
Gain	11.35%	12.71%	28.96%	17.11%

MPI ranks/ joules	20	40	80
Double	14437	11686	10612
Mixed	13403	10509	8033
Gain	7.16%	10.07%	24.30%

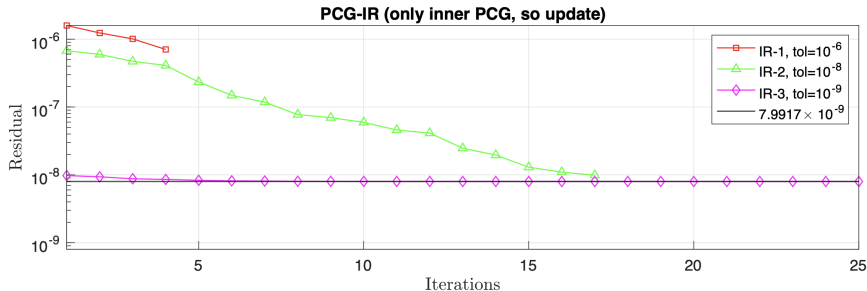
- ▶ Solving the Poisson's equation with Neko
- ▶ Time is for the CG loop
- ▶ Energy measurements are for the entire code

Work in progress: PCG with iterative refinement

- 1: PCG to solve $Ax_m = b$ until $tol = 10^{-4}$ ▷ FP32. PCG breaks at 10^{-6}
- 2: **for** $i \leftarrow 1, 2, 3$ **do** ▷ Run for few iterations
- 3: $r = b - Ax_m$ ▷ FP64
- 4: $r_{new} = r / \|r\|$ ▷ FP64
- 5: PCG to solve $Ad_{new} = r_{new}$ until $tol = 10^{-6}, 10^{-8}$ ▷ FP32
- 6: $d = d_{new} * \|r\|$ ▷ FP64
- 7: $x_m = x_m + d$ ▷ FP64
- 8: **end for**

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Summary

- ▶ Computer arithmetic operates with **finite precisions**
- ▶ Use **computer arithmetic tools** to
 - ▶ detect cancellations
 - ▶ get the right FP format
 - ▶ verify sensitivity of reduced precision
- ▶ Enabling mixed-precision in CFD codes:
 - ▶ use tools: Verificarlo, gprof, Intel Advisor
 - ▶ target the most time-consuming parts
 - ▶ **reduced time-to-solution by up to 40 % and energy-to-solution by up to 2x** on MareNostrum5 & LUMI

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Thank you for your attention !

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References

- ▶ Iakymchuk et al. *Best Practice Guide – Harvesting energy consumption on European HPC systems: Sharing Experience from the CEEC project*. Zenodo, Aug 2024
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