Reliable and sustainable computations: An application-driven approach

Roman Iakymchuk[1]
joint work with Pablo Oliveira (Paris-Saclay)

Umeå University, Sweden
riakymch@cs.umu.se

ISC, Hamburg, Germany
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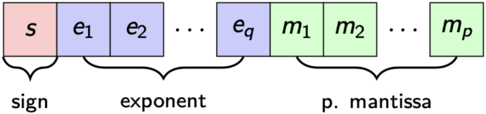
Outline

1. Floating point arithmetic
2. Reliable and sustainable computations
3. Analysing applications' precision demands with tools

Floating-point IEEE-754 representation

IEEE-754 defines a standardized FP representation

$$f = s \times 2^e \times m$$



$$
\begin{aligned}
(1.1001 \times 2^0)_2 =& (1 \times 2^0 + 1 \times 2^{-1} + 0 \times 2^{-2} + 0 \times 2^{-3} + 1 \times 2^{-4})_{10} \\
=& (1 + 0.5 + 0.125)_{10} = 1.625_{10}
\end{aligned}
$$

- FP64 (double): 1 (sign) / 11 (exp.) / 52 (mantissa)
- FP32 (double): 1 (sign) / 8 (exp.) / 23 (mantissa)

Reducing precision

IEEE-754

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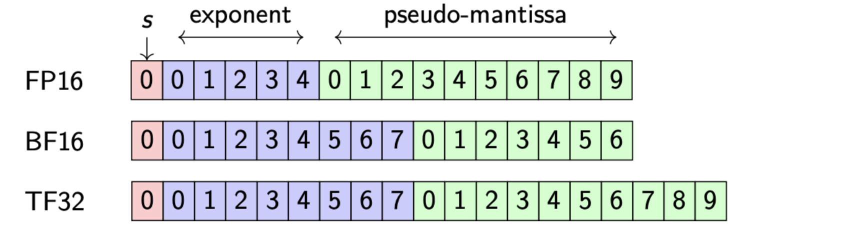
- FP32 (double): 1 (sign) / 8 (exp.) / 23 (mantissa)

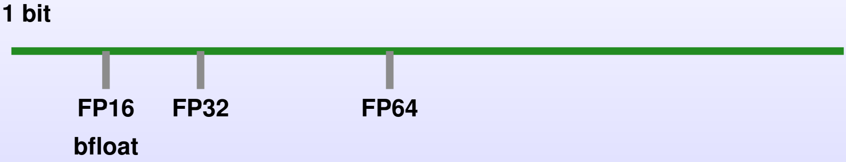
- Shift from FP64 towards high throughput smaller datatypes
  - Strong trend driven by AI workloads
  - Neural network using lower-precision TF32, FP16 and BF16
  - Gain in throughput performance and energy efficiency



Mixing precisions

- double-single plus iterative refinement

Mixing precisions



- double-single plus iterative refinement
- double-single-half/ bfloat
- over 100 works on mixed precision [a]

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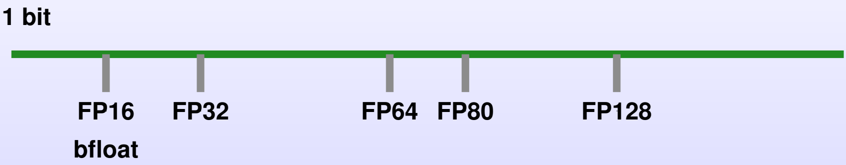
[a] Nicholas Higham and Théo Mary. 'Mixed Precision Algorithms in Numerical Linear Algebra'

Mixing precisions



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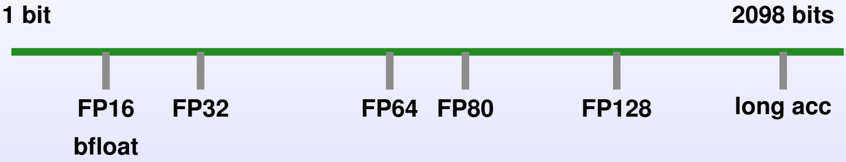
Mixing precisions



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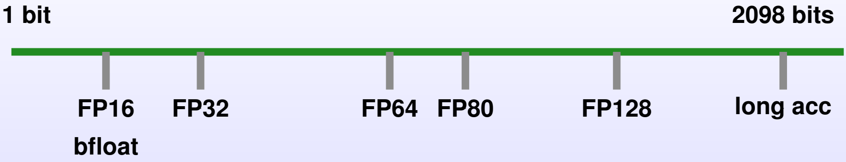
  - floating-point expansion with error free transformation (`twoprod` & `twosum`)
  - long accumulator

Mixing precisions



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  - long accumulator
- **Mixed-precision algorithm** is an algorithm that reliably and effectively combines multiple precisions and techniques

$$\boxed{Ax = b}$$

while $(\tau > \tau_{\max})$

Step	Operation	Kernel	Communication
$S1:$	$w := Ad$	SPMV	Allgatherv*
$S2:$	$\rho := \beta / < d, w >$	DOT product	Allreduce
$S3:$	$x := x + \rho d$	AXPY	–
$S4:$	$r := r - \rho w$	AXPY	–
$S5:$	$z := M^{-1} r$	Apply preconditioner	–
$S6:$	$\beta := < z, r >$	DOT product	Allreduce
$S7:$	$d := (\beta / \beta_{old}) d + z$	AXPY-like	–
$S8:$	$\tau := < r, r >$	DOT product	Allreduce

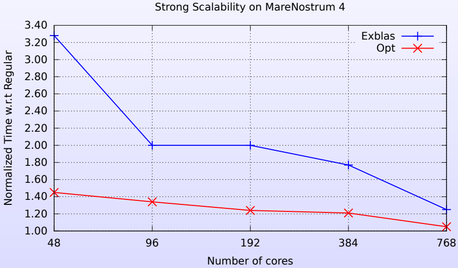
end while

Robustness of Algorithms

- **Robustness: accuracy and reproducibility**
- FP ops are non-associative : $(-1+1)+2^{-53} \neq -1+(1+2^{-53})$
- Non-reproducibility in PCG: `dot`, `axpy`, and `spmv`

→ Solution : ExBLAS (ParCo15, NRE15, JCAM, IJHPCA)

3D Poisson equation with 27 stencil points and $tol = 10^{-8}$



Iteration	Residual			
	MPFR	*Original* 1 proc	*Original* 48 procs	*Exblas & Opt*
0	0x1.19f179eb7f032p+49	0x1.19f179eb7f03**3**p+49	0x1.19f179eb7f03**3**p+49	0x1.19f179eb7f032p+49
2	0x1.f86089ece9f75p+38	0x1.f86089**f08810d**p+38	0x1.f86089e**d07a76**p+38	0x1.f86089ece9f75p+38
9	0x1.fc59a29d329ffp+28	0x1.fc59a29d**1b6a**p+28	0x1.fc59a29d**2e989**p+28	0x1.fc59a29d329ffp+28
10	0x1.74f5ccc211471p+22	0x1.74f5cc**b8203ad**p+22	0x1.74f5ccc**1fafef**p+22	0x1.74f5ccc211471p+22
...	...	...	...	...
40	0x1.7031058eb2e3ep-19	0x1.703105**aea0e8a**p-19	0x1.7031058e**8ff5a**p-19	0x1.7031058eb2e3ep-19
42	0x1.4828f76bd68afp-23	0x1.4828f**6fabbf2a**p-23	0x1.4828f76b**b9038**p-23	0x1.4828f76bd68afp-23
45	0x1.8646260a70678p-26	0x1.8646260**1300d2**p-26	0x1.8646260a7**1301**p-26	0x1.8646260a70678p-26
47	0x1.13fa97e2419c7p-33	0x1.13fa9**8038c44e**p-33	0x1.13fa97e**54e903**p-33	0x1.13fa97e2419c7p-33

Table 3: Accuracy and reproducibility comparison on the intermediate and final residual against MPFR for a matrix with condition number of 10^{12}. The matrix is generated following the procedure from Section 5.1 with n=4,019,679 (159^3).

Sustainable algorithms

Idea

lagom - not too much, not too little, *just the right amount*

Sustainable algorithms

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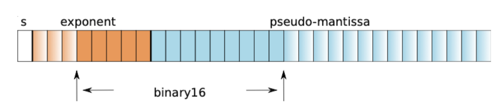
lagom - not too much, not too little, *just the right amount*

Analysis w tools $\rightarrow$ Strategy $\rightarrow$ Revision of algorithms

1. **arithmetic tools** applied to **codes** $\rightarrow$ manual / automatic
2. if the reduction is possible, apply algorithmic solutions
3. conduct probabilistic (aka optimistic) error analysis
   - error bound with constant $\sqrt{n}\mu$ with high probability
4. implement on hardware with stochastic rounding support – randomly maps x to one of two bounds

- Verificarlo – an automatic tool for debugging and assessing FP precision based on Monte Carlo Arithmetic
- Backends: debugging (MCA) and mixed-precision (Vprec)
- Eg setting $r = 5$ and $p = 10$, VPREC simulates a `binary16` embedded inside a `binary32`



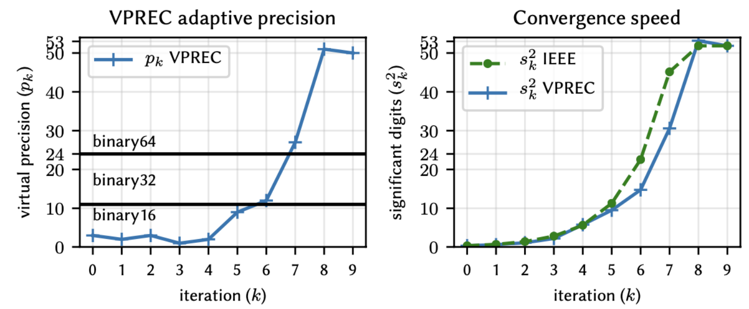
k	x_{k+1}	s_k^{10}	s_k^2
0	0.0690266447076745	0.11	0.37
1	0.1230846130203958	0.21	0.70
2	0.1985746566605835	0.43	1.43
3	0.2732703639721015	0.84	2.79
4	0.3119369815109966	1.79	5.95
5	0.3181822938100336	3.40	11.3
6	0.3183098350392471	6.79	22.6
7	0.3183098861837825	13.6	45.2
8	0.3183098861837907	15.6	51.8
9	0.3183098861837907	15.6	51.8

```
double newton(double x0) {
  double x_k, x_k1=x0, b=PI;
  do {
    x_k  = x_k1;
    x_k1 = x_k*(2-b*x_k);
  }while (fabs((x_k1-x_k)/x_k)
  >= 1e-15);
  return x_k1;
}
```

The Newton-Raphson method for inverse of π[a]

[a] Pablo Oliveira et al. *Automatic exploration of reduced floating-point representations in iterative methods.* Euro-Par 2019

VerifiCarlo-Vprec Example



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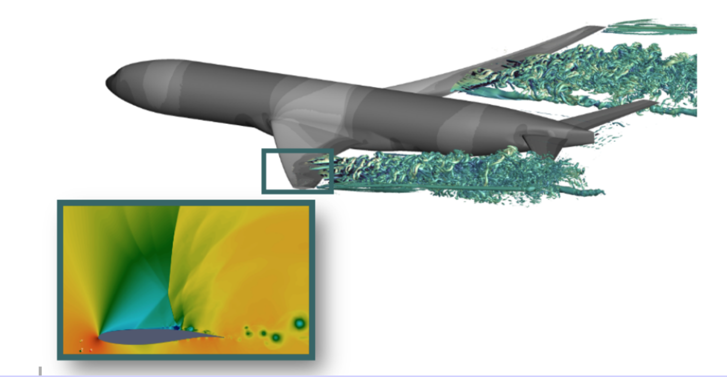
Center of Excellence for Exascale CFD



- **Sustainable algorithmic solutions with mixed-precision and tools**
- **Consortium codes**: Neko, NEK5000/ NekRS, FLEXI, waLBerla

Nekbone test case

- **NEK5000** is a high order, incompressible Navier-Stokes solver based on the spectral element method
- **Nekbone** solves a Poisson equation using a Conjugate Gradient method with a simple or spectral element multigrid preconditioner
- **AMG** benchmark – parallel algebraic multigrid solver for linear systems arising from problems on unstructured grids
  - Two solvers: CG and GMRES

Nekbone w Vprec

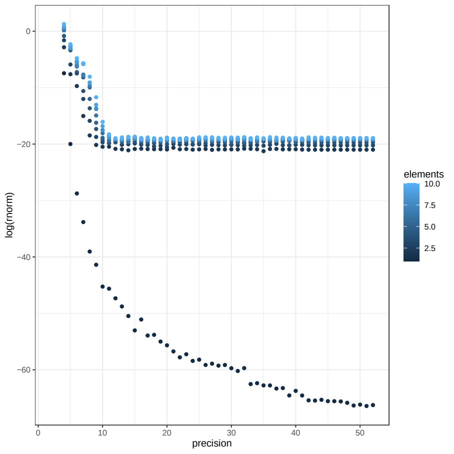
Basic example w/o preconditioner

$$\boxed{Ax = b}$$

while $(\tau > \tau_{\max})$

Step	Operation
$S1:$	$w := Ad$
$S2:$	$\rho := \beta / < d, w >$
$S3:$	$x := x + \rho d$
$S4:$	$r := r - \rho w$
$S5:$	$z := M^{-1} r$
$S6:$	$\beta := < z, r >$
$S7:$	$d := (\beta / \beta_{old}) d + z$
$S8:$	$\tau := < r, r >$

end while



Nekbone w `Vprec`

Multigrid Preconditioner Example

$$\boxed{Ax = b}$$

while $(\tau > \tau_{\max})$

Step	Operation
$S1:$	$w := Ad$
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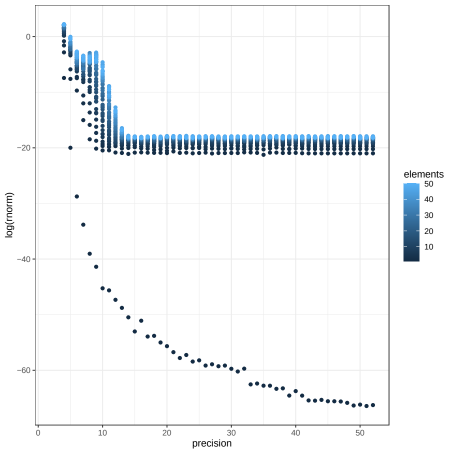
end while



- **20 MCA samples** for the previously found VPREC configuration
- simulate `binary32`

$$\boxed{Ax = b}$$

while $(\tau > \tau_{\max})$

Step	Operation
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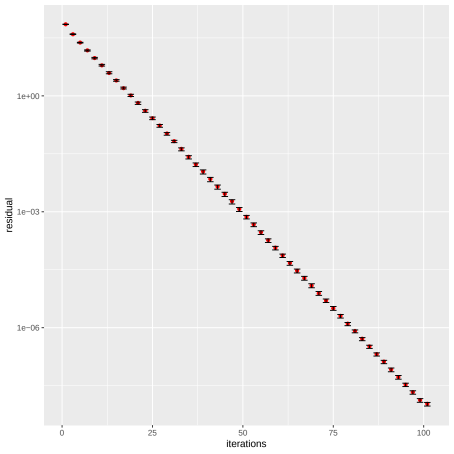
$S8:$	$\tau := < r, r >$

end while



Summary

- mixed-precisions is a way toward sustainable computing
- employ computer arithmetic tools for
  - numerical abnormalities detection
  - precision requirements inspection
  - numerical CI etc

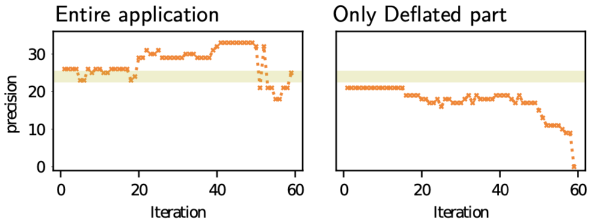


Figure: Minimal precision that preserves convergence.

Yales2 CFD application with the DPCG solver: 16 % energy gain

Automatic exploration of reduced floating-point representations in iterative methods.

Chatelain, Petit, de Oliveira Castro, Lartigue, Defour. Euro-Par 2019

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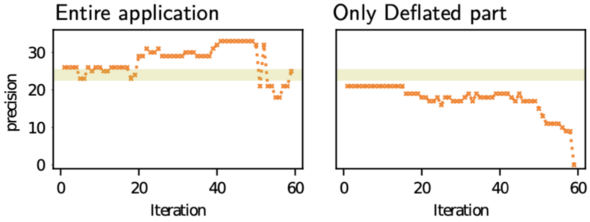

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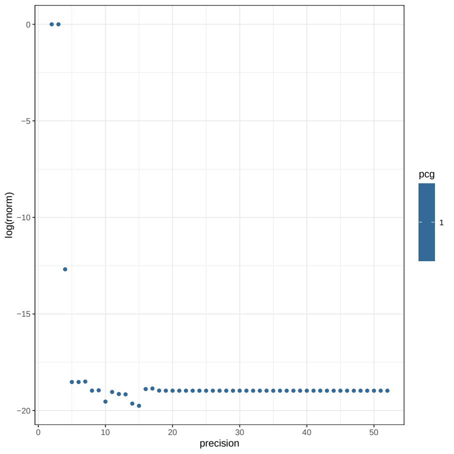
Thank you for your attention !

AMG w Vprec

AMG-PCG
Laplace type problem



AMG-GMRES
Non-linear time-dependent problem